CS2305.501

Fall 2015

Homework 2

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5 points each part, except as noted.

1.4.8 a-d

a.) For every animal x, if the animal is a rabbit then the animal hops.

b.) For every animal x, the animal is a rabbit and the animal hops.

c.) For some animal x, if the animal is a rabbit then the animal hops.

d.) For some animal x, the animal is a rabbit and the animal hops.

1.4.10

a.) ∃xS(x) (C(x) ∧ D(x) ∧ F(x))

b.) ∀xS(x) (C(x) ∧ D(x) ∧ F(x))

c.) ∃xS(x) (C(x) ∧ F(x) ∧ ¬D(x))

d.) ¬∀xS(x) (C(x) ∧ D(x) ∧ F(x))

e.) ∃xS(x) (C(x) ∧ D(x) ∧ F(x))

1.5.4 a-f

a.) There exists some student x that has taken some computer science class y.

b.) There exists some student x that has taken all computer science class y.

c.) All students x have taken some computer science class y.

d.) There exists some computer science class y that all students x have taken.

e.) All computer science classes y have been taken by some student x.

f.) All students x have taken all computer science classes y.

1.5.8 a-d

a.) ∃x∃yQ(x) (x,y)

b.) ¬∀x∃yQ(x) (x,y)

c.) ∃x∃yQ(x) (x, Jeopardy ∧ Wheel of Fortune)

d.) ∀y∃xQ(x) (x,y)

e.)

1.6.18

We know that some x exists such that x is shorter than y. We can choose Max as the y value, however we cannot be sure what the corresponding x value is.

1.6.24

Line 7 has two universal quantifiers distributing to Q(x)

1.6.28

1.) ∀x(P(x) ∨ Q(x)) Premise

2.) P(a) ∨ Q(a) Universal Instantiation using (1)

3.) Q(a) Disjunctive Syllogism (2)

4.) ∀x(¬P(x) ∧ Q(x) → R(x)) Premise

5.) (¬P(a) ∧ Q(a) → R(a)) Universal Instantiation using (4)

6.) (¬P(a) ∧ T → R(a)) Substitution (5)

7.) (¬P(a) → R(a)) Identity (6)

8.) P(a) Disjunctive Syllogism (2)

9.) P(a) → ¬R(a) Modus Tollens (7)

10.) ∀x( P(x) → ¬R(x)) Universal Generalization (8)

(You need to prove a universally quanti\_ed statement - how can you do that? As a subgoal,

you will need to prove an implication - how can you do that?)

1.7.6 (15 points)

1.) Assume x and y are odd

2.) x = 2k +1 Definition of odd (k is an integer)

3.) y = 2l + 1 Definition of odd (l is an integer)

4.) xy = (2k + 1) (2l +1) Substitution

5.) xy = 4kl + 2k + 2l +1 Distribution

6.) xy = 2(2kl + k + l) + 1 Factoring

7.) m = 2kl + k + 1 Definition for substitution

8.) xy = 2(m) + 1 Sub. m (integers closed under multiplication)

9.) xy = odd Definition of odd

1.7.26 (15 points)

Direct proof

1.) Assume n is a positive integer

2.) Assume n is even

3.) 7n + 4 Premise

4.) 7(2k) + 4 Definition of even (k is an integer)

5.) 14k + 4 Substitution, k closed under multiplication

6.) 2(7k+2) Factoring

7.) m = 7k + 2 Definition for substitution, m int. closed under +

8.) 2(m) Sub. m (integers closed under multiplication)

9.) 7n + 4 = even Definition of even

10.) If n is even, 7n +4 is even (1-9)

Proof by contrapositive of converse

1.) Assume n is a positive integer

2.) Assume n is odd Contrapositive of converse

3.) 7n + 4 Premise

4.) 7(2k+1) + 4 Definition of odd (k is an integer)

5.) 14k +11 Distribution, k closed under multiplication

6.) 14k +10 +1 Rewrite

7.) 2(7k+5) + 1 Factor

8.) m = 7k + 5 Definition for substitution, m integer closed by +

9.) 2(m)+ 1 Sub. m (integers closed under multiplication)

10.) 7n + 4 = odd Definition of off

11.) If n is odd, 7n +4 is odd (1-10)

Both proofs together

1.) If n is even, 7n+4 is even

2.) If n is odd, 7n+4 is odd

3.) Therefore, n is even if and only if 7n+4 is even.